All solutions to graded problems must be typed in LaTeX. All other problems can be typed or written, but must be completed. Graded problems must be organized in numerical order. All problem sets (both graded and non-graded) must be uploaded to Canvas as a single PDF document. Students can find the grading scale in our Syllabus.

## Graded

**Exercise 1.** Let $C$ be a relation on $A$ and let $A_0 \subset A$. Define the *restriction* of $C$ to $A_0$ by the relation $C \cap (A_0 \times A_0)$.

1. Show that the restriction of an equivalence relation is an equivalence relation.

2. Show that the restriction of an order relation is an order relation.

**Exercise 2.** Prove the following theorem:

**Theorem.** If an ordered set $A$ has the least upper bound property, then it has the greatest lower bound property.

## Non-graded

**Exercise 3.** Define a relation $<$ on the plane $\mathbb{R} \times \mathbb{R}$ by setting

$$(x_0, y_0) < (x_1, y_1) \text{ if } \begin{cases} x_0^2 + y_0^2 < x_1^2 + y_1^2 \text{ or,} \\ x_0^2 + y_0^2 = x_1^2 + y_1^2 \text{ and } x_0 < x_1 \text{ or,} \\ x_0^2 + y_0^2 = x_1^2 + y_1^2 \text{ and } x_0 = x_1 \text{ and } y_0 < y_1 \end{cases}$$

Show that this relation is an order relation on the plane.

**Exercise 4.** Let $A = \mathbb{R} \times \mathbb{R} - \{(0,0)\}$, the plane with origin removed. Define two points $(x_0, y_0), (x_1, y_1) \in A$ to be equivalent if there exists a real number $t$ such that $(x_0, y_0) = (tx_1, ty_1)$. Check that this is an equivalence relation and describe the equivalence classes.